TRACES OF ANISOTROPIC FUNCTION SPACES. APPLICATIONS

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1. INTRODUCTION

Let R_2 be the two-dimensional euclidean space, the plane. Let $1 < p < \infty$ and $s = 1,2,3,\ldots$. Then

(1) $$W_p^s(R_2) = \{f \mid f \in L_p(R_2),\ \|f|W_p^s(R_2)\| = \sum_{|\alpha| \leq s} \|D^\alpha f| L_p(R_2)\| < \infty\}$$

are the classical Sobolev spaces, where $L_p(R_2)$ has the usual meaning (complex-valued functions). It is well-known that the trace-operator

(2) $$R : f(x_1,x_2) \to f(x_1,0)$$

is a retraction from $W_p^s(R_2)$ onto the special Besov space ($\sim$ Lipschitz space) $B_p^{s-1/p}(R_1)$ on the real line R_1 . Here *retraction* means that there exists a linear and bounded extension operator S from $B_p^{s-1/p}(R_1)$ (the trace space) into $W_p^s(R_2)$ such that

(3) $$RS = id \quad (\text{identity in } B_p^{s-1/p}(R_1)\ ) .$$

In other words, if a trace-operator is a retraction then this assertion covers both the "direct" and the "inverse" embedding theorems and indicates that R is a mapping "onto". The above-mentioned special Besov spaces $B_p^\sigma(R_1)$ with $\sigma > 0$ and $1 < p < \infty$ are defined as follows. If $t \in R_1$ and $\tau \in R_1$ then

(4) $$(\Delta_\tau^1 f)(t) = f(t+\tau) - f(t) \ , \qquad \Delta_\tau^m = \Delta_\tau^1\, \Delta_\tau^{m-1} \ ,$$

with $m = 2,3,\ldots$ are the usual differences. Then $B_p^\sigma(R_1)$ is the collection of all complex-valued functions such that