THE MALLIAVIN CALCULUS AND LONG TIME ASYMPTOTICS OF CERTAIN WIENER INTEGRALS

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1. INTRODUCTION

The asymptotic behavior of stochastic oscillatory integrals has recently received much attention in the probabilistic literatures and is closely related to various problems in the analysis and applied mathematics, (cf. [3],[4],[6],[5],[10],[14]~[17] and [19]). In particular, in order to study asymptotic properties of stochastic oscillatory integrals, Malliavin [17] has used the stochastic calculus of variation. Gaveau and Moulinier [5] have also been interested in similar problems. The main purpose of this paper is to complete in detail the proof of Malliavin's results which was sketched in [17]. To do this, as is shown in the section 5, we need some considerations which are not discussed in [17], (see Propositions 5.1 and 5.2). We will also give a slight extension of some results in Malliavin [17].

Let us consider a smooth Riemannian metric g on R^d which is uniformly elliptic and bounded, (see Assumption 2.2). Then there exists the diffusion process $\{X(t), P_x, x \in R^d\}$ generated by half the Laplace-Beltrami operator $\Delta_g/2$ with respect to g. For every smooth differential 1-form θ, we set

$$K_t(x,y;\theta) = E_x[\exp\{\sqrt{-1}\int_{X[0,t]} \theta\} | X(t) = y] \tag{1.1}$$

$$\text{for} \quad (t,x,y) \in (0,\infty) \times R^d \times R^d$$