BOUNDARY VALUE PROBLEMS OF LINEAR ELASTOSTATICS AND HYDROSTATICS ON LIPSCHITZ DOMAINS

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Section 1 Introduction

In this note I will report on some recent progress in the study of boundary value problems for systems of equations on Lipschitz domains D in $\mathbb{R}^n$, with boundary data in $L^2(\partial D, d\sigma)$. The specific problems I will discuss here arise from elastostatics and hydrostatics.

The Dirichlet problem for a single equation (the Laplacian) on a Lipschitz domain D with $L^2(\partial D, d\sigma)$ data and optimal estimates was first treated by B. E. J. Dahlberg (see [3], [4], and [5]). His approach relied on positivity, Harnack's inequality and the maximum principle, and thus, it could not be used to study for example the Neumann problem, or systems of equations. Shortly afterwards, E. Fabes, M. Jodeit, Jr., and N. Riviere [6] were able to utilize A. P. Calderon's ([1]) theorem on the boundedness of the Cauchy integral on C^1 curves, to extend the classical method of layer potentials to the case of C^1 domains. In this work they were able to resolve the Dirichlet and Neumann problem with $L^2(\partial D, d\sigma)$ data, and to obtain optimal estimates, for C^1 domains. They relied on the Fredholm theory, exploiting the compactness of the layer potentials in the C^1 case. In 1979, D. Jerison and C. Kenig [9] were able to give a simplified proof of Dalhberg's results, using an integral identity that goes back to Rellich ([15]). However, the method still relied on positivity. Shortly afterwards, they were also able to treat the Neumann problem on