THE REGULARITY OF WEAK SOLUTIONS TO PARABOLIC SYSTEMS

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There has been a lot of work devoted to the regularity of weak solutions to elliptic systems since De Giorgi [3] had shown in 1968 that his celebrated regularity result for elliptic equations cannot be extended to systems. Various methods such as the direct approach, the indirect approach and the hole-filling technique were developed to study the partial and everywhere regularity for quasilinear and nonlinear elliptic systems (see e.g. [4], [8] and the references cited there). It is reasonable to ask the following questions. How about the problem of regularity for parabolic systems? Are the results for elliptic systems still true for them? And do the methods mentioned above work in the parabolic case? Basically, the answers to these questions are positive.

Let Ω be an open set in $\mathbb{R}^n$, $T > 0$ and $Q = \Omega \times (0,T)$. Denote $z = (x,t)$, where $x \in \mathbb{R}^n$, $t \in \mathbb{R}$. For $z_i = (x_i,t_i) \in \mathbb{R}^{n+1}$, $i = 1,2$, introduce the parabolic metric

$$d(z_1,z_2) = |x_1-x_2| + |t_1-t_2|^{\frac{1}{2}} .$$

Throughout this paper we use the convention that repeated indices are to be summed for α and β from 1 to n , for i and j from 1 to N , but not for k .