BEST CONTINUITY AT THE BOUNDARY FOR SOLUTIONS OF THE MINIMAL SURFACE EQUATION

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We consider the Dirichlet problem for the minimal surface equation. We assume that Ω is a bounded open set in $\Re^n$ and ϕ is a continuous function defined on $\partial\Omega$. Then we consider the problem:

Find $u \in C^2(\Omega) \cap C^0(\overline{\Omega})$ such that

(i) $u = \phi$ on $\partial\Omega$,

(ii) u satisfies the minimal surface equation in Ω, that is,

$$\sum_{i=1}^{n} D_i\left(\frac{D_i u}{\sqrt{1+|Du|^2}}\right) = 0 \qquad \text{in} \qquad \Omega.$$

Additionally we assume that $\partial\Omega$ is C^2 and has nonnegative mean curvature, H, everywhere. This assumption means that there is a unique solution for the problem ([JS]). We examine the way the regularity of u depends on the regularity of ϕ.

If $k \geq 2$, $0 < \alpha < 1$ and $\phi, \partial\Omega \in C^{k,\alpha}$ then the estimates of Jenkins and Serrin [JS] plus standard theory ([GT]) show that $u \in C^{k,\alpha}(\overline{\Omega})$. The case $k = 1$ has been considered by Lieberman [L] and also Giaquinta and Giusti [GG], and they showed that if $\phi \in C^{1,\alpha}(\partial\Omega)$ then $u \in C^{1,\alpha}(\overline{\Omega})$. The case $k = 0$ was studied by Giusti [G] and also by the author [W1], [W2], [W3]. The results of Giusti, from 1972, may be summarized as follows: