# REGULARITY OF THE SINGULAR SETS IN IMMISCIBLE FLUID INTERFACES AND SOLUTIONS TO OTHER PLATEAU-TYPE PROBLEMS

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Existence and almost everywhere regularity of solutions to a wide variety of Plateau-type problems follows from several geometric measure theory theorems (due primarily to DeGiorgi, Federer, Fleming, Reifenberg, Almgren, and Allard.) But singularities do occur, and the size and nature of the singular set depends strongly on the particular problem. In this paper we describe our recent discovery that, for many problems, the singular sets are also fairly regular. The pioneering work on regularity of singular sets was done by Jean Taylor [T1,T2] for certain two-dimensional surfaces in $\mathbb{R}^3$; our work is a simplification and generalization of hers.

An important ingredient in the proofs is a slight extension of a stratification of singularities theorem due to Almgren [A2,2.27]. The theorem says that an m-dimensional surface S that is stationary for any of the Plateau-type problems we consider stratifies naturally as

$$S = \cup_{i=0}^{m} \Sigma_i$$

where $\Sigma_i$ has Hausdorff dimension $\leq i$. The stratification is by tangent cone type. In particular, $\Sigma_m$ consists of all points at which each tangent cone is an m-plane, and $\Sigma_{m-1}$ consists of all points at which each tangent cone is a union of half-planes meeting along an (m-2) dimensional subspace. (The lower levels of the stratification need not concern us here.) In the particular problems we consider, it