CLASSIFICATION OF MINIMIZING HYPERSURFACES ASYMPTOTIC TO QUADRATIC CONES IN $\mathbb{R}^{n+1}$

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A celebrated theorem of S. Bernstein states that in $\mathbb{R}^3$, every entire minimal graph is an affine plane [B]. Nowadays, Bernstein's theorem can be understood as a corollary to a much broader result, sometimes called the *parametric Bernstein theorem*: *when* $n \leq 7$, *every area-minimizing hypersurface in* $\mathbb{R}^{n+1}$ *is an affine hyperplane.* Here, *hypersurface* means a current S of the form

$$S = \partial[\![U]\!],$$

where $[\![U]\!]$ denotes the current corresponding to oriented integration of $(n+1)$-forms over an open set $U \subset \mathbb{R}^{n+1}$. We say that S is *area-minimizing* if, whenever $r > 0$ and $B_r = \{x \in \mathbb{R}^{n+1} : |x| < r\}$, we have

$$\|S\|B_r \leq \|S+Z\|B_r$$

for all hypersurfaces Z supported within B_r. (i.e., inside B_r, S has less n-area than any other hypersurface which agrees with it outside B_r.) In particular, a Standard Stokes' Theorem argument (see e.g. [SL3]) immediately shows that entire minimal graphs are area-minimizing; this is why the original theorem of Bernstein follows from the later parametric version.

Here, we concern ourselves with the question of what happens when the parametric Bernstein theorem fails, as it does when $n \geq 8$. This