EXISTENCE OF MINIMAL SURFACES OF BOUNDED TOPOLOGICAL TYPE IN THREE-MANIFOLDS

Jon T. Pitts and J.H. Rubinstein*

In this paper we describe various recent results on the existence of minimal surfaces in three-manifolds. In a variety of contexts, we are able to establish the existence of smooth, embedded, two-dimensional, minimal submanifolds of three-manifolds, where the genus and index of instability of the minimal surface are bounded independently of the metric on the three-manifold. In one version of our theorem, we obtain closed minimal surfaces in compact three-manifolds; in a second version, we obtain minimal surfaces with boundary lying in the boundary of a uniformly convex subset of $\mathbb{R}^3$. As a consequence of our theorems, we are able to obtain a number of new examples in which minimal surfaces are realized in three-manifolds in topologically interesting ways. The details will appear in [PR]. Our methods are those of geometric measure theory.

We now describe our results and methods in more detail. We begin with several definitions. The *genus* of a compact, two dimensional, topological manifold (with or without boundary) is defined to be the number of handles in the manifold if it is orientable, and the number of cross caps if it is not orientable. Let Σ be a smooth, compact, connected, oriented, three dimensional, Riemannian manifold with Heegard genus H. The *Heegard genus of* Σ is the least genus for which there is a smooth, compact, connected, embedded, two dimensional submanifold M of Σ of that genus such that $\Sigma \sim M$ has exactly two connected components, each of which is a handlebody. Such an M is called a *Heegard surface in* Σ. M is necessarily orientable.

As is well known [SJ], whenever S is a smooth, compact, embedded, two dimensional, minimal submanifold (with or without boundary) of Σ, there exists a

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