A NOTE ON BRANCHED STABLE TWO-DIMENSIONAL MINIMAL SURFACES

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§1. INTRODUCTION AND STATEMENT OF RESULT

In [3], D. Fischer-Colbrie and R. Schoen investigate the properties of stable minimal surfaces in 3-manifolds of non-negative scalar curvature. As a special case, they prove that, the only complete, stable, oriented, immersed minimal surfaces in $\mathbb{R}^3$ are planes, a result which was also proved by do Carmo and Peng [2]. For applications, it is useful to know whether this theorem still holds if the minimal surface has branch points (an argument given in §3 shows that the usual second variation of area formula is valid even in the presence of branch points). The answer, in general, is no: there exist branched minimal surfaces in $\mathbb{R}^3$ of the conformal type of the disk whose Gauss image lies in a disk of arbitrarily small radius in S^2 (see, for instance, [5, page 73]); these are then stable by a theorem in [1] (see also Remark 5 in [3]). In this note we show, however, that if, as a Riemann surface, the minimal surface is of parabolic type (i.e. it admits no non-constant positive superharmonic functions), the stability of the minimal surface in $\mathbb{R}^3$ implies that it is a plane. This is actually a special case of the following

THEOREM. *Let* $F : M^2 \to \mathbb{R}^4$ *be a (possibly branched) stable minimal immersion of an open oriented surface* M^2 *without boundary. Then* F *is holomorphic with respect to an orthogonal complex structure on* $\mathbb{R}^4$ *if any one of the following conditions holds:*

(i) M *is parabolic in the conformal structure induced by* F