THE TOPOLOGY OF ASYMPTOTICALLY EUCLIDEAN STATIC PERFECT FLUID SPACE-TIME

A.K.M. Masood-ul-Alam

1. INTRODUCTION

In this paper we prove that a geodesically complete, asymptotically Euclidean, static perfect fluid space-time having a connected fluid region and satisfying the time-like convergence condition is diffeomorphic to $\mathbb{R}^3 \times \mathbb{R}$. It is believed that such a space-time would be spherically symmetric at least for physically reasonable conditions on the density function ρ and the pressure function p. The above assertion (that the space-time is diffeomorphic to $\mathbb{R}^3 \times \mathbb{R}$) has been claimed in [1] provided the Poincaré conjecture is valid. In fact a theorem due to Gannon [2] says that such a space-time is diffeomorphic to $N \times \mathbb{R}$ where N is a simply connected complete 3-manifold. The asymptotic conditions then imply that N has the same homotopy as $\mathbb{R}^3$ ([1]). Thus Gannon's result reduced the question to proving the non-existence of fake 3-cells in N. In particular it would give the full result if the 3 dimensional Poincaré conjecture were known to be true.

2. STATIC PERFECT FLUID SPACE-TIME

By a static perfect fluid spacetime we mean a geodesically complete space-time $(M, {}^4g)$ such that:

(i) M is a C^∞ manifold diffeomorphic to $N \times \mathbb{R}$ where for each $t \in \mathbb{R}$, $N_t = N \times \{t\}$ is a spacelike three-manifold.