WHEN ARE SINGULAR INTEGRAL OPERATORS BOUNDED?

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The aim of this talk is to survey some results concerning the L_2-boundedness of singular integral operators, and in particular to present the $T(b)$ theorem.

Let us consider one-dimensional singular integral operators T of the following type:

$$(Tu)(x) = \text{p.v.} \int_{-\infty}^{\infty} K(x,y)u(y)dy$$

where, for $x,y \in \mathbb{R}$ with $x \neq y$,

$$(1) \qquad \begin{cases} |K(x,y)| \le c_0 |x-y|^{-1} \\ \left|\frac{\partial K}{\partial x}(x,y)\right| \le c_1 |x-y|^{-2} \\ \left|\frac{\partial K}{\partial y}(x,y)\right| \le c_2 |x-y|^{-2} . \end{cases}$$

Such T are called Calderón-Zygmund operators if $\|T\varphi\|_2 \le c\|\varphi\|_2$ for all $\varphi \in C_0^{\infty}(\mathbb{R})$. We note first that an L_2-estimate of this type is sufficient to prove a variety of bounds.

THEOREM 1 (Calderón, Zygmund, Cotlar, Stein) *Suppose* T *is a Calderón-Zygmund operator. If* $u \in L_p$, $1 < p < \infty$, *then* $Tu(x)$ *is defined for almost all* x, *and* $\|Tu\|_p \le c_p\|u\|_p$, $1 < p < \infty$. *If*