THE NEUMANN PROBLEM FOR EQUATIONS OF MONGE-AMPÈRE TYPE

P-L. Lions

N.S. Trudinger

J.I.E. Urbas

In the paper [10] we are concerned with the existence of classical solutions to the semilinear Neumann problem for equations of Monge-Ampère type

$$\text{(1)} \qquad \det D^2u = f(x,u,Du)$$

in convex domains Ω in Euclidean n-space, $\mathbb{R}^n$, where f is a prescribed positive function on $\bar{\Omega}\times\mathbb{R}\times\mathbb{R}^n$. In conjunction with (1), we treat Neumann boundary conditions of the form

$$\text{(2)} \qquad D_\nu u = \varphi(x,u)$$

on the boundary $\partial\Omega$, where ν denotes the unit inner normal on $\partial\Omega$ and φ is a given function on $\partial\Omega\times\mathbb{R}$. For the main existence theorem, whose statement follows, we assume that Ω is uniformly convex with boundary $\partial\Omega \in C^{3,1}$, $f \in C^{1,1}(\bar{\Omega}\times\mathbb{R}\times\mathbb{R}^n)$ is positive and non-decreasing in z, for all $(x,z,p) \in \bar{\Omega}\times\mathbb{R}\times\mathbb{R}^n$, and $\varphi \in C^{2,1}(\partial\Omega\times\mathbb{R})$ is non-decreasing in z with

$$\text{(3)} \qquad \varphi_z(x,z) \geq \gamma_0$$