BOUNDARY BEHAVIOR OF SOLUTIONS OF ELLIPTIC EQUATIONS IN "BAD" DOMAINS

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The natural setting for the theory of nondivergence form second order elliptic equations is in the Hölder spaces $C^{k,\alpha}$. To explain this statement, consider the elliptic operator Δ, the Laplacian. Then, the map $u \to \Delta u$ is a bijection of $C^{2,\alpha}(\bar{\Omega})$ onto $C^{\alpha}(\bar{\Omega})$ provided the boundary values of u are fixed and $\partial\Omega \in C^{2,\alpha}$; however, this map is not a bijection of $C^2(\bar{\Omega})$ onto $C^0(\bar{\Omega})$ because it is never surjective. (We do not consider the mapping from $W^{2,p}(\Omega)$ to $L^p(\Omega)$ because the appropriate boundary conditions cannot be described intrinsically via the same sort of spaces.)

To pin down the boundary values, we consider the Dirichlet boundary condition,

(1) $$u = u_0 \quad \text{on} \quad \partial\Omega$$

for some $u_0 \in C^{2,\alpha}(\partial\Omega)$, and the oblique boundary condition

(2a) $$\beta \cdot Du = g \quad \text{on} \quad \partial\Omega$$

for some vector field $\beta \in C^{1,\alpha}(\partial\Omega)$ satisfying

(2b) $$\beta \cdot \gamma > 0 \quad \text{on} \quad \partial\Omega,$$

where γ is the inner normal, and $g \in C^{1,\alpha}(\partial\Omega)$. With these boundary conditions, we ask how much the regularity of u_0, β, g, and $\partial\Omega$ can be relaxed without losing the desirable feature that the boundary condition still