DEFORMING RIEMANNIAN METRICS ON THE 2-SPHERE

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In 1982, Hamilton [Ha] proved the following:

Theorem Let X be a compact 3-dimensional Riemannian manifold of positive Ricci curvature. The evolution equation $\frac{\partial}{\partial t} g_{ij} = \frac{2}{3} r g_{ij} - 2R_{ij}$, where $r = \int_X R d\mu_X / \int_X d\mu_X$, has a unique solution for all t and it converges as $t \to \infty$ to a metric of constant positive curvature. Furthermore, any isometries of X are preserved as the metric evolves.

The aim of this paper is to prove a 2-dimensional version of this theorem. We have also obtained analogous results for Kähler and Hermitian manifolds by applying the same method with Huisken's higher dimensional version of Hamilton's theorem [Hu].

We start with a compact, oriented Riemannian surface of positive Gaussian curvature (already this is enough to show that M is diffeomorphic to S^2 by the Gauss-Bonnet theorem and the classification of compact surfaces). We then show that there is a principal S^1 bundle over M with a metric of positive Ricci curvature such that the projection map is a Riemannian submersion. We allow the metric on this bundle to evolve to a metric of constant curvature; the metric on M then evolves to a metric of constant curvature also.