BOUNDARY REGULARITY FOR SOLUTIONS OF QUASI-LINEAR ELLIPTIC EQUATIONS

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1. INTRODUCTION

We consider the boundary regularity of a classical solution $u(x) \in C^0(\bar{\Omega}) \cap C^2(\Omega)$ to the Dirichlet problem of a class of quasi-linear elliptic equations:

$$(1.1) \qquad \begin{aligned} Q(u) \equiv a_{ij}(x,u,Du)D_{ij}u &= 0 \qquad \text{in } \Omega , \\ u &= \varphi \qquad \text{on } \partial\Omega , \end{aligned}$$

where Ω is a bounded C^2 domain in $\mathbb{R}^n$, $n \geq 2$ and $\varphi \in C^0(\partial\Omega)$ has some modulus of continuity β . Here we use the usual summation convention for repeated indices.

We refer to [GT], [JS] for the case when $\varphi \in C^{2,\alpha}(\partial\Omega)$, [GG], [G], [Li 1] for $\varphi \in C^{1,\alpha}(\partial\Omega)$, [Li 3] for φ having $D\varphi$ Dini continuous and [Li 2], [Sl] for $\varphi \in C^{0,1}(\partial\Omega)$.

We shall mainly discuss how the order of non-uniformity (h) and the geometry (convexity) of Ω affect the regularity of a solution of (1.1) near the boundary. As was remarked in [B], when $0 \leq h < 1$, the operator Q behaves very similarly to the Laplace operator (where $h = 0$) ; when $1 \leq h \leq 2$, some convexity (or some generalized convexity) condition has to be imposed on Ω . A typical representative of the latter class is the minimal surface operator (where $h = 2$) . Since this is discussed in