A PIECEWISE LINEAR THEORY OF MINIMAL SURFACES IN 3-MANIFOLDS

William Jaco[1] *and J.H. Rubinstein*[2]

Introduction

In an impressive series of papers, Meeks and Yau [MYi, 1 ≤ i ≤ 5], Meeks, Simon and Yau [MSY], Freedman, Hass and Scott [FHS], Scott [S], and Meeks and Scott [MS] introduced and used least area surfaces in the investigation of topological problems about 3-manifolds. This has lead to the solution of many outstanding questions in the topology of 3-manifolds. An example is the positive solution of the Smith conjecture (see [SC]), in which the results of Meeks and Yau [MY5] played an important role.

In [JR1], we used least weight normal surfaces to obtain the equivariant decomposition theorems of 3-manifolds in [MYi, 1 ≤ i ≤ 5] and [MSY]. These least weight normal surfaces share many of the same useful properties as least area surfaces. However since the Meeks-Yau exchange and roundoff trick cannot be directly applied to normal surfaces, we were unable to recapture the more difficult applications and properties of least area surfaces in [S], [MS] and [FHS], by using least weight normal surfaces.

Here we develop the idea of least weight normal surfaces to obtain piece-wise linear (PL) minimal surfaces in 3-manifolds. This theory has several advantages over the classical area of analytic minimal surfaces, especially with regard to the study of the topology of 3-manifolds. Firstly, to establish existence of PL minimal surfaces, there is no necessity to appeal to deep results from partial differential equations and geometric measure theory, as in the analytic case. (See Hass-Scott [HS] for a new uniform treatment of existence theory for least area surfaces, using only Morrey's solution of Plateau's problem in Riemannian 3-manifolds). For PL minimal surfaces, it suffices to use the short classical PL technique of Kneser [K], plus a little elementary analysis.