# MINIMAL SURFACES WITH FREE BOUNDARIES

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We will in this lecture give a survey of some recent results for minimal surfaces with free boundaries. To this end, we consider boundary configurations $\langle \Gamma, S\rangle$ in $\mathbb{R}^3$ consisting of *a fixed part* $\Gamma$ and a *free part* S . The fixed part $\Gamma$ is the union of Jordan arcs $\Gamma_1,\ldots,\Gamma_m$ , and the free part consists of surfaces $S_1,\ldots, S_n$ in $\mathbb{R}^3$ with or without boundary. Each of the curves $\Gamma_j$ is either a closed curve, or else an arc with end points on S . In the following, the fixed part $\Gamma$ may be void, whereas the free part S is always assumed to be non-empty.

A mapping $X : \bar{\Omega} \to \mathbb{R}^3$ of some Riemann surface $\Omega$ with boundary $\partial\Omega$ into $\mathbb{R}^3$ is called a *solution of the free boundary problem for the configuration* $\langle \Gamma, S\rangle$ if the following properties are satisfied:

(i) $X \in C^0(\bar{\Omega},\mathbb{R}^3) \cap C^2(\Omega,\mathbb{R}^3)$ ;

(ii) X is a harmonic mapping ;

(iii) X maps $\Omega$ conformally onto $X(\Omega)$ , except for isolated branch points in $\Omega$ ;

(iv) $X(\partial\Omega) \subset \Gamma \cup S$ ;

(v) the surface $\mathcal{M} = X(\bar{\Omega})$ intersects S at $\Sigma \cap \text{int } S$ perpendicularly. Here $\Sigma$ denotes the *free trace* $X(\partial\Omega)$ of the minimal surface $\mathcal{M}$ on the free boundary S.

Obviously, property (v) does not make sense since we assumed X to be only continuous at the boundary $\partial\Omega$. Therefore, (v) has to be understood in a weak sense. However, it follows from well known