DEGREE THEORY AND GAUSSIAN MEASURES IN INFINITE DIMENSIONS

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In this talk, I will explain a new approach to the definition of the degree of a map between infinite dimensional spaces. First, recall the definition of the Leroy-Schauder degree, as given by Elworthy and Tromba [2]:

Let $F : B \to B$ be a nonlinear map on the Banach space B such that

i) F is compact, in the sense that $d_x F$ is a compact linear map on B for all $x \in B$;

ii) $I + F$ is a proper map.

Then the degree of F is defined to be

$$\deg(F) = \sum_{x+F(x)=y} \operatorname{sgn}(I + d_x F) ,$$

where y is any regular value of the map $I + F$; that is $I + d_x F$ is invertible for all x such that $x + F(x) = y$. (The number $\operatorname{sgn}(I + d_x F) = \pm 1$, according to whether the linear map $I + d_x F$ preserves or reverses the orientation of B .) The basic result of the theory is that this definition of degree is independent of the regular value $y \in B$ that is used, and that the set of regular values of $I + F$ is generic, and hence, non-empty.

Consider the following map, from the space of C^α maps from the circle S to $\mathbb{R}^N$ into the space of $C^{\alpha-1}$ maps from the circle to $\mathbb{R}^N$:

$$f(t) \mapsto \frac{df(t)}{dt} + (\nabla V)(f(t)) ,$$

where V is a C^∞ real function on $\mathbb{R}^N$, and hence ∇V is a C^∞