PARTIAL REGULARITY FOR SOLUTIONS OF VARIATIONAL PROBLEMS*

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We report here on some recent results of the authors [FH1,2] within the context of a general discussion of problems in the calculus of variations. Some results (those in [FH2]) were not included in the delivered lecture.

We will consider minima of functionals $\mathcal{F}$ of the form

$$u \mapsto \mathcal{F}[u] = \int_{\Omega} F(x,u,Du) \tag{1}$$

where $\Omega \subset \mathbb{R}^n$, Ω is open, and $u : \Omega \to \mathbb{R}^N$. It will always be assumed that F is a *Caratheodory* function, i.e. $F = F(x,u,p)$ is measurable in x for all $(u,p) \in \mathbb{R}^n \times \mathbb{R}^N$ and is continuous in (u,p) for almost all $x \in \Omega$. This ensures that $F(x,u,Du)$ is measurable if u is measurable.

Here will be interested in the general case $n \geq 1$ and $N \geq 1$. If $N = 1$, one can obtain much stronger results, for this we refer to [G1], [G2], [GT], [LU], and [M].

There are two questions of fundamental interest. First, one wants to show (subject to various boundary conditions) the *existence* of minima of $\mathcal{F}$ in suitable function classes. Second, one is interested in the *regularity* (i.e. smoothness) properties of such minimisers.

The existence problem in a general sense is solved as a standard consequence of the following result by Acerbi and Fusco [AF].

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* Lecture delivered by the second author.