# ABOUT SOME ILL-POSED PROBLEMS

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## 1. INTRODUCTION

The purpose of this note is to give an informal account of recent results, obtained jointly with A. Friedman [1,2] about some non-linear ill-posed problems arising in fluid dynamics.

Before describing them, we wish to make some remarks about ill-posed problems, as an attempt to single out the mathematical issues we are interested in.

There is not a "canonical" definition of ill-posed problems; they are a set of facts, diverse in scope and motivation, and are difficult to unify in a group of issues and methods. Efforts towards an organic theory have been made only in recent years. We mention in particular the books of Tikhonov-Arsenin [17] and Lavrentiev [9]; the monograph of L. Payne [10], and a beautiful review article of G. Talenti [16].

The following way of looking at ill-posed problems is partial and incomplete; however it is adequate for the purpose of this note.

A boundary value problem associated with a partial differential equation is ill-posed if

(a) a solution can be found (if at all!) only for a very narrow class of data, or

(b) if a solution exists, it exhibits instabilities with respect to small variations of the data.

The meaning of "narrow class of data", "instabilities", "small variation", has to be made precise in each single problem, through the specification of appropriate spaces and topologies.