THE GAUSS MAP OF A SUBMERSION

Paul Baird

1. MOTIVATION

Let $\phi: (M, g) \to (N, h)$ be a mapping of Riemannian manifolds. Then, following Eells and Sampson [11], we let τ_ϕ denote the tension field of the map ϕ. Thus τ_ϕ is a section of the pull-back bundle $\phi^{-1}TN$, and ϕ is <u>harmonic</u> if and only if $\tau_\phi = 0$ [9].

The fundamental problem of harmonic mappings is, given a particular homotopy class of mappings between Riemannian manifolds; does it contain a harmonic representative. In the case when M and N are surfaces a complete picture is known. For example, consider the homotopy classes of maps from the torus (T^2, g) to the sphere (S^2, h), with any metrics. All classes with degree $|d| \geq 2$ have harmonic representatives. The classes with $d = \pm 1$ have no harmonic representatives [9]. In the case when M and N have arbitrary dimensions we have a much less complete picture.

A natural case to study is when M and N are spheres. Many examples of harmonic mappings were constructed by Smith [17] by assuming certain symmetry. He showed in particular that all classes of $\pi_n(S^n)$ have a harmonic representative for $n \leq 7$. In [1] this work was carried further and it was shown by allowing ellipsoidal deformations of the metrics that all classes of $\pi_n(S^n)$ for any n have a harmonic representative at least in some metric.

Another case considered by Smith were the classes $\pi_3(S^2) \cong Z$. The homotopy classes of maps from S^3 to S^2 are parametrized by their Hopf