ASYMPTOTIC BEHAVIOUR NEAR ISOLATED SINGULAR POINTS FOR GEOMETRIC VARIATIONAL PROBLEMS

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Isolated singularities for extrema of functionals of "geometric" type have been studied in [SL1], [SL2]. Here we will use the notation of [SL1]. We consider functionals of the form

$$(1)\qquad \mathcal{F}(u) = \int_0^\infty \int_\Sigma e^{-mt}\Big[F(\omega,u,\nabla u,\frac{\partial u}{\partial t}) + E(\omega,t,u,\nabla u,\frac{\partial u}{\partial t})\Big]d\omega dt$$

where Σ is a compact manifold, m constant $\neq 0$, ∇ = gradient on Σ , and where E has exponential decay with respect to t as $t \uparrow \infty$. Here, u is a C^2 section of a vector bundle V over $\Sigma \times (0,\infty)$.

For these functionals, it is proved in [SL1] that under certain conditions, e.g. that the C^2 norm of u on $\Sigma \times (0,\infty)$ is finite, $F(\omega,z,p,q)$ convex in p , $q\cdot F_q(\omega,z,p,q) \geq |q|^2$ for $|q| \leq 1$ and $q\cdot F_q(\omega,z,p,q) > 0$ for $q \neq 0$, and $F(\omega,z,p,0)$ real analytic in (z,p) , an extremum u of (1) has a limit as $t \uparrow \infty$. However, the method of proof does not yield estimates for the *rate* of convergence to the limit, except in special circumstances.

We also consider the functionals

$$\mathcal{F}_\Sigma(u) = \int_\Sigma F(\omega,u,\nabla u,0)d\omega \ .$$