THE OBLIQUE DERIVATIVE PROBLEM FOR EQUATIONS OF MONGE-AMPERE TYPE IN TWO DIMENSIONS

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1. Introduction

In this paper we are concerned with the existence of convex classical solutions of the oblique derivative problem for equations of Monge-Ampère type,

(1.1) $$\det D^2u = f(x,u,Du) \text{ in } \Omega ,$$

(1.2) $$D_\beta u = \varphi(x,u) \text{ on } \partial\Omega .$$

Here Ω is a uniformly convex domain in $\mathbb{R}^2$, f , φ are prescribed functions on $\bar{\Omega} \times \mathbb{R} \times \mathbb{R}^2$, $\partial\Omega \times \mathbb{R}$ respectively with f positive, and β is a unit vector field on $\partial\Omega$ satisfying

(1.3) $$\beta\cdot\nu > 0 ,$$

where ν is the inner unit normal to $\partial\Omega$. This problem, and in particular the case $\beta = \nu$, was recently studied for domains $\Omega \subset \mathbb{R}^n$, $n \geq 2$, by Lions, Trudinger and Urbas [5], who proved a priori estimates for the derivatives up to second order for convex solutions of (1.1), (1.2) under suitable regularity and structure hypotheses on Ω , f , φ and β . In particular, the second derivative estimate in [5] requires $\beta = \nu$, and it does not appear that the method used there can be modified to work for more general β . However, if the domain Ω is a ball and $f^{1/n}$ is convex with respect to the gradient variables,