APPLICATIONS OF MINIMAX TO MINIMAL SURFACES AND THE TOPOLOGY OF 3-MANIFOLDS

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In this paper, we describe some recent constructions of and applications of minimal surfaces in 3-manifolds. These minimal surfaces arise from a minimum/maximum construction developed in [PR2] (and summarized in detail in [PR1]). (See also [PJ1] and [SS] for earlier versions of the method.) An outline of the basic procedure is as follows:

Let Σ be a closed connected oriented Riemannian 3-manifold and suppose Λ is an oriented Heegard surface in Σ; i.e., the closures of the two components of $\Sigma \sim \Lambda$ are handlebodies K and K'. We consider one-parameter smooth families $\Lambda_t, t \in [0,1]$, sweeping out Σ and having these properties: Λ_0 and Λ_1 are graphs; Λ_t is isotopic to Λ for all $0 < t < 1$; the handlebody K_t for $0 < t < 1$ is chosen so that the orientation on K_t coming from Σ induces the given orientation on Λ_t; and K_t converges to Λ_0 as $t \to 0+$ and the limit of K_t as $t \to 1-$ is Σ. The fundamental theorem is the following.

THEOREM 1. *There are a sequence of families Λ_t^i and choices of parameter t_i so that as $i \to \infty$, $\Lambda_{t_i}^i$ converges (in the $\mathbf{F}$ metric for varifolds) to a smooth closed embedded minimal surface M such that* $\mathrm{genus}(M) \leq \mathrm{genus}(\Lambda)$ *and* $\mathrm{index}(M) \leq 1 \leq \mathrm{index}(M) + \mathrm{nullity}(M)$.

REMARKS. (1) Theorem 1 has turned out to be a versatile and powerful tool. In this paper we describe six interesting applications. In practice, applying Theorem 1 is comparatively straightforward. In a given situation, where one seeks minimal surfaces with specific properties, it typically suffices to specify a single (non-minimaxing)

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