ON THE BEHAVIOUR OF SOLUTIONS TO A SEMILINEAR NEUMANN PROBLEM

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§1. Introduction

In this expository paper we wish to survey some recent results on the following semilinear Neumann problem (with the diffusion coefficient d varied as a parameter)

$$(1.1)\qquad \begin{cases} d\Delta u - u + u^p = 0 & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded smooth domain in $\mathbb{R}^n$, ν denotes the unit outer normal to $\partial\Omega$, $\Delta \equiv \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ and $d > 0$, $p > 1$ are two constants. Equation (1.1) arises naturally in various models in mathematical biology; for instance, it is equivalent to an elliptic chemotaxis system and is also known as the "shadow" system of an activator-inhibitor system of Gierer and Meinhardt.

Chemotaxis is the oriented movement of cells in response to chemicals in their environment. For example, cellular slime molds (amoebae) release a certain chemical, move toward places of its higher concentration and then form aggregates. In 1970 Keller and Segel [KS] proposed a model, which in particular includes the following system (see e.g. [S]), to describe the chemotactic aggregation stage of cellular slime molds: