ON REMOVABLE ISOLATED SINGULARITIES OF SOLUTIONS TO A CLASS OF QUASI-LINEAR ELLIPTIC EQUATIONS

Chi-ping Lau

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1. INTRODUCTION

Let Ω be some open subset of $\mathbb{R}^N$ containing 0 and $\Omega' = \Omega \sim \{0\}$. Let u be a solution of

$$-\Delta u + u|u|^{q-1} = 0 \qquad \text{in } \Omega' . \tag{1.1}$$

Brezis and Véron [2] proved that u can be extended to be a solution of (1.1) in all of Ω if $q \geq N/(N - 2)$, $N \geq 3$. Hence isolated singularities of (1.1) are "removable". Véron [8] showed that the exponent $N/(N - 2)$ is the best possible because there exist singular solutions when $1 < q < N/(N - 2)$. Aviles [1] generalized the result in [2] by replacing the Laplacian by some linear operators in divergence form. Vazquez and Véron showed that we can also replace the Laplacian by the quasi-linear p-Laplacian $\operatorname{div}(|Du|^{p-2}Du)$, $N > p > 1$. Here $Du = (D_1u, \ldots, D_Nu)$ denotes the gradient of the function of u .

A natural question is to ask whether the Laplacian can be replaced by a more general class of quasi-linear elliptic operators which include the above mentioned examples.