SOME REGULARITY THEORY FOR CURVATURE VARIFOLDS

John E. Hutchinson

Suppose M is a smooth n-dimensional manifold in $\mathbb{R}^N$ and for each $x \in M$ let $P(x)$ be the matrix of the orthogonal projection of $\mathbb{R}^N$ onto T_xM. Then the second fundamental form is morally given by the following N^3-tuple

$$A = A_{ijk} = (\nabla^M P_{jk})_i , \tag{1}$$

where $1 \leq i,j,k \leq N$ (the usual version of the second fundamental form is easily computable from A, and conversely, see [2]).

More generally, suppose V is an n-dimensional varifold in $\mathbb{R}^N$. In other words, V is a Radon measure on $\mathbb{R}^N \times G(n,N)$, where $G(n,N)$ is the set of all orthogonal projections of $\mathbb{R}^N$ onto some n-dimensional subspace and is naturally imbedded in $\mathbb{R}^{N^2}$. Then we say $A = [A_{ijk}]_{1 \leq i,j,k \leq N}$ is the weak second fundamental form of V if

$$\text{(a) } A_{ijk} \in L^1_{loc}(V) \text{ for } i,j,k \leq N \tag{2}$$

$$\text{(b) } \int \left\{ P_{ij} \frac{\partial}{\partial x_j} \phi(x,P) + A_{ijk}(x,P) \frac{\partial}{\partial P_{jk}} \phi(x,P) + A_{jij}\, \phi(x,P) \right\} dV(x,P) = 0$$

for all $\phi \in C_1(x_1,\dots,x_N,P_{11},\dots,P_{NN})$ which are compactly supported in $x_1,\dots,x_n$.