REGULARITY AND SINGULARITY FOR ENERGY MINIMIZING MAPS

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1. INTRODUCTION

We will consider the occurrence of singularities in a class of boundary-value mapping problems. Suppose M is an m dimensional smooth compact Riemannian manifold with boundary and N is a smooth compact Riemannian manifold without boundary. Via an isometric embedding, we view N as a Riemannian submanifold of $\mathbb{R}^k$. We will consider the following type of problem:

Given a smooth $\varphi : \partial M \to N$, *find a least energy* $u : M \to N$ *with* $u \mid \partial M = \varphi$.

While various general energy functionals may be treated, we will mainly discuss, for $1 < p < \infty$, the ordinary *p-energy*

$$\int_M |\nabla u|^p \, dM \quad .$$

Here, the most important case is $p = 2$ where critical points are *harmonic maps.* In local coordinates $x_1, x_2, \cdots, x_m$ on M, the expression $|\nabla u|^p$ should be interpreted as $[\Sigma_{\alpha,\beta} \Sigma_{i,j} (\partial u^i/\partial x_\alpha) g^{\alpha,\beta} (\partial u^j/\partial x_\beta)]^{p/2}$ and the volume element dM as $(\det g)^{1/2} dx$ where $g = g_{\alpha,\beta} = [g^{\alpha,\beta}]^{-1}$ is the matrix representing the metric of M in these coordinates. Since only the topology and geometry of N will be relevant for our discussion of regularity and singularity, we will, for simplicity of notations, assume that M *is an open subset of* $\mathbb{R}^m$ *with the standard Euclidean metric.*

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