MAXIMAL SURFACES AND GENERAL RELATIVITY

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In this report I'd like to review the development of the regularity theory of maximal (and prescribed) mean curvature hypersurfaces and describe some of the main ideas that are involved. Such surfaces have long been an important tool in general relativity, but it is only recently that their regularity properties have been fully described [CY], [BS], [G], [B1], [B2]. These results, culminating in [B2] which showed that variational extremal hypersurfaces are smooth and spacelike, are described in some detail in the following sections; here I will give a brief summary of the main properties and applications of mean curvature hypersurfaces in general relativity, together with selected references.

Perhaps the most important reason for the utility of constant mean curvature hypersurfaces is their uniqueness property [BF], [G]; in a cosmological spacetime — i.e. a spatially compact, globally hyperbolic Lorentzian manifold satisfying the *timelike convergence condition*

$$\text{(TCC)} \qquad \mathrm{Ric}(T,T) \geq 0 \quad \text{for all timelike vectors } T \quad ,$$

a constant (non-zero) mean curvature Cauchy surface is unique [BF] and a maximal (i.e. zero mean curvature) Cauchy surface is almost unique [G]. Thus constant mean curvature slicings provide a "canonical" choice of global time function in a cosmological spacetime, and maximal surfaces parameterised by the "time at infinity" play a similar role in asymptotically flat spacetimes [B1].