REGULARITY OF GENERALIZED SCALAR OPERATORS WITH SPECTRUM CONTAINED IN A LINE

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(Dedicated to Professor Igor Kluvánek)

Generalized scalar operators were introduced by C. Foiaş [4] and a detailed study of such operators can be found in the monograph [3]. An important subclass consists of regular generalized scalar operators which enjoy properties not shared by all generalized scalar operators. For example, the sum and product of commuting generalized scalar operators S and T need not be generalized scalar operators[2; §3]. However, if, in addition, S and T are both regular, then ST and S + T are again generalized scalar operators [3; p.106], although they need not be regular [2; §3]. In particular, there exist generalized scalar operators which are not regular [1,2].

A closed subset F of the complex plane $\mathbb{C}$ is called *thin* [3; p.100] if the function $\lambda \to \overline{\lambda}$ on F (the bar denotes complex conjugation) is the restriction of a function which is analytic in a neighbourhood of F. It is clear that any closed subset of a thin set is also a thin set and that segments of a line are thin sets. Accordingly, the following result is an immediate consequence of Theorem 4.1.11 in [3].

THEOREM. *A generalized scalar operator whose spectrum is contained in a line in the complex plane is necessarily regular.*

The proof of this result given in [3] is based on the theory of distributions and N. Dunford's analytic functional calculus. The purpose of this note is to present another proof of the above Theorem based on some recent work of A. McIntosh and A. Pryde [6] in which they develop a specific functional