INTEGRATION OF CONUCLEAR SPACE VALUED FUNCTIONS

(dedicated to Professor Igor Kluvánek)

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In the classical theory of the Lebesgue integral, the space of integrable functions is introduced as the completion of the space of simple functions (or of continuous functions), with respect to the topology of convergence in mean, that is, the uniform convergence of indefinite integrals. However, if the space of simple functions taking values in a Banach space X is considered, then its completion can only be realized as the space of integrable functions with values in some locally convex space larger than X (see [6]).

In this note, it is shown that the space of integrable functions with values in a conuclear space Y is sequentially complete with respect to the topology of convergence in mean. Further, the Y-valued simple functions form a dense linear subspace. In this case, there is no necessity to integrate functions taking values in a locally convex space larger than Y. The class of conuclear spaces includes the spaces $\mathcal{D}$, $\mathcal{D}'$, $\mathcal{E}$, $\mathcal{E}'$, $\mathcal{S}$ and $\mathcal{S}'$ which arise in distribution theory, as well as the product space and the locally convex direct sum of countably many copies of the real line.