ON THE EXISTENCE OF POSITIVE ENTIRE SOLUTIONS OF NONLINEAR ELLIPTIC PROBLEMS IN R^N

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Consider the elliptic problem

(1) $$Lu = f(x,u), \quad x \in R^N$$

(2) $$u \in C^2(R^N), \quad \lim_{|x|\to\infty} u(x) = 0,$$

for $N \geq 3$, where

$$Lu = - \sum_{i,j=1}^{N} \frac{\partial}{\partial x_i} (a_{ij}(x)\frac{\partial u}{\partial x_j});$$

each $a_{ij} \in C^{1+\alpha}_{loc}(R^N)$, $0 < \alpha < 1$, the matrix $(a_{ij}(x))$ is bounded and uniformly positive definite in R^N, and the conditions (A) below hold.

Our main objective is to prove the existence of positive solutions of (1), (2), and obtain asymptotic estimates. A prototype of this class is the equation

(3) $$- \Delta u = p(x)u^{\gamma}, \quad x \in R^N,$$

$1 < \gamma < \frac{N+2}{N-2}$, and $p(x) \not\equiv 0$ is a locally Hölder continuous function in R^N, satisfying $0 \leq p(x) \leq C(1 + |x|^2)^{-b}$ for some constants C, $1 < b < N/2$. The problem has been the subject of intensive investigations in recent years. In particular, there are several results on the existence of positive solutions of equation (1) which are bounded below by positive constants, see, for example, [5], [6], and the