# SOME BASIC SEQUENCES AND THEIR MOMENT OPERATORS

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## 1. INTRODUCTION

A well known result in Fourier analysis (see [3, p.107], for example) says that if the Fourier series of a continuous function on the circle group is lacunary, then the series converges uniformly to the function. Equivalently, if $(\alpha(n))$ is a lacunary sequence of positive integers (that is, $\alpha(n+1)\alpha(n)^{-1} \geq \gamma > 1$, for all $n$ and some $\gamma$), then the sequence $1, e^{i\alpha(1)t}, e^{-i\alpha(1)t}, e^{i\alpha(2)t}, e^{-i\alpha(2)t}, \ldots$ is basic in $C(0, 2\pi)$.

On the other hand, Gurarii and Macaev ([5]) proved some analogues of this result for power sequences in $C([0,1])$ and $L^p(0,1)$. Letting $1 \leq p < \infty$ and letting $(\alpha(n))$ be a given increasing sequence of positive numbers, they proved that $(\alpha(n))$ is lacunary if and only if $(\alpha(n)^{1/p}t^{\alpha(n)-1/p})$ is basic in $L^p(0,1)$, in which case this basic sequence is equivalent to the standard basis in $\ell^p$. They also proved that $(\alpha(n))$ is lacunary if and only if $(t^{\alpha(n)})$ is basic in $C([0,1])$.

In [4], Edwards has considered, in a dual form, a related problem concerning sequences of measures on a compact Hausdorff space $K$. If $(\mu_n)$ is a weak* convergent sequence of measures on $K$ which satsifies a one term recurrence relation, he gives conditions which ensure that $\{(\int_K f d\mu_n) : f \in C(K)\} = c$. This result is closely related to the problem of finding conditions for $(\mu_n)$ to be a basic sequence of measures on $K$.

The present paper presents some analogues of the preceding results which are derived by considering a general problem in Banach spaces. Throughout, $X$ will denote a given Banach space with dual $X^*$, $(b_n)$ will denote a given sequence of scalars, $\sigma = (v_n)$ will denote a given sequence of vectors in $X$ and $\tau = (x_n)$ will denote the sequence in $X$

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