# ASYMPTOTIC LIMITS IN MULTI-PHASE SYSTEMS

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In this note we consider the asymptotic behaviour of an inviscid fluid with heat conduction. This work has been done in conjunction with J. Ball [2]. The fluid is assumed homogeneous and to occupy a spatial region $\omega \subset R^n$, where $\omega$ is bounded and open. At time t and position $x \in \omega$ the fluid has density $\rho(x,t) \geq 0$, velocity $v(x,t) \in R^n$, and temperature $\theta(x,t) > 0$. For simplicity we assume there is no external body force or heat supply. The governing equations are then

$$\rho\dot{v} = - \text{grad } p \tag{1}$$

$$\dot{\rho} + \rho \, \text{div}(v) = 0 \tag{2}$$

$$\rho\dot{U} + \rho \, \text{div}(v) + \text{div}(q) = 0 \tag{3}$$

where the dots denote material time derivatives, p is the pressure, U the internal energy density and q the (spatial) heat flux vector. The constitutive relations are given in terms of the Helmholtz free energy, $A(\rho,\theta)$ and specific entropy $\eta(\rho,\theta)$, by

$$p = \rho^2 \frac{\partial A}{\partial \rho} \quad , \quad \eta = - \frac{\partial A}{\partial \theta} \quad , \quad U = A + \eta\theta \tag{4}$$

$$q = q(\rho,\theta,\text{grad } \theta).$$

We impose the boundary conditions

$$v \cdot n \Big|_{\partial\omega} = 0 \tag{5}$$