BILINEAR INTEGRALS AND RADON-NIKODYM DERIVATIVES

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Dedicated to Igor Kluvánek

1. INTRODUCTION

A basic problem in the treatment of random evolutions via the non-commutative Feynman-Kac formula is to construct a random multiplicative operator valued functional M, and show that the perturbed semigroup of the random evolution is represented as the integral of M with respect to the associated operator valued measure [3]. The solutions of certain partial differential equations can thereby be represented as a bilinear integral, giving further insight into their behaviour.

A related problem is to determine when a process described by an operator valued measure may be expressed as the integral of an operator valued function with respect to another such process.

The purpose of this paper is to consider the above question in a general setting. Given measures m, n with values in the vector spaces Y, Z respectively, there is a given continuous bilinear "multiplication" defined on the product of the vector spaces X and Y with values in Z, and we ask whether n can be expressed as the integral of an X- valued function with respect to m .

The present approach differs from previous work [1], [6] in a number of ways. The locally convex setting is used, because ultimately, the case of spaces of operators on Banach spaces, with the strong operator topology is to be treated. The bilinear multiplication - the composition of operators - is only separately continuous in that case.

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