EIGEN-EXPANSIONS OF SOME SCHRÖDINGER OPERATORS AND NILPOTENT LIE GROUPS

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This note is a summary of the results previously obtained by the authors, and of a number of new results and problems.

In papers [8] – [10] the authors studied Schrödinger operators $H = L + V$, on $\mathbb{R}^d$,.there

(1) $$-L = \sum_{j=1}^{d} (-1)^{n_j} D_j^{2n_j} , \quad n_j \geq 1$$

(2) $$V(x) = \sum_{j=1}^{k} P_j^2(x) , \text{ where } P_j \text{ are real polynomials.}$$

We say that the family of polynomials is *irreducible* if there is no linear change of variables in $\mathbb{R}^d$, such that all the polynomials depend on a smaller number of variables. By restriction to a lower dimensional subspace of $\mathbb{R}^d$, we consider only operators H for which the polynomials $P_1,\dots,P_k$ are irreducible.

THEOREM 1 [9] *For some* N *and* $\lambda > 0$, $(\lambda + H)^{-N}$ *is a Hilbert-Schmidt operator on* $L^2(\mathbb{R}^d)$.

Let $0 < \lambda_1 \leq \lambda_2 \leq \dots$ be the eigenvalues and $\varphi_1,\varphi_2,\dots$ the corresponding eigen-functions of H .

THEOREM 2 [9] *There is an* N *such that if* $K \in C^N(\mathbb{R}^+)$ *and*

(3) $$(1 + \lambda)^N |K^{(j)}(\lambda)| \leq C \quad \textit{for} \quad \lambda > 0 , \; j = 0,\dots,N$$