ON A MODIFIED MEAN CURVATURE FLOW

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In recent years various evolution equations have been studied in differential geometry. In a landmark paper Hamilton [4] studied the deformation of Riemannian metrics on compact manifolds in direction of their Ricci-curvature:

(1)
$$\frac{d}{dt}g_{ij} = -2Ric_{ij} + \frac{2}{n}rg_{ij}$$

Here g_{ij} is the Riemannian metric, Ric_{ij} is the Ricci curvature and $r = \int \operatorname{Rd} \mu$ is the average of the scalar curvature on the manifold. This is a weakly parabolic system and Hamilton showed that on a three dimensional manifold any initial metric of positive Ricci-curvature flows into a metric of constant positive curvature when evolved by equation (1). This result was later extended to higher dimensions in [5] and [7].

In [6] we studied the mean curvature flow, that is an evolution equation for hypersurfaces M^n embedded in \mathbb{R}^{n+1} : Let the initial hypersurface M^n_0 be given locally by some diffeomorphism

$$F_0: U \subset \mathbb{R}^n \to F_0(U) \subset \mathbb{M}_0 \subset \mathbb{R}^{n+1} ,$$

then we want to find a whole family F(.,t) of diffeomorphisms corresponding to hypersurfaces M_{\pm} such that the evolution equation

$$\frac{d}{dt} F(\vec{x}, t) = -H(\vec{x}, t) . v(\vec{x}, t) \qquad \vec{x} \in U$$

(2)

 $F(.,0) = F_0$