THE FIRST BOUNDARY VALUE PROBLEM FOR SOLUTIONS OF DEGENERATE QUASILINEAR PARABOLIC EQUATIONS

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The study of a class of degenerate quasilinear parabolic equations such as $u_t = (u^m)_{xx} (m>1)$; $u_t = [a(u)]_{xx} + [b(u)]_x$, $a'(0) = 0$; $u_t = \Delta(u^m)(m>1)$; $u_t = \Delta a(u) + \Sigma b_i(u) u_{x_i}$, $a'(0) = 0$ etc. comes from liquid flow through porous medium. Attention is restricted to the nonnegative solutions because of their meaning in this context. A special feature of the above equations is, that at the point (x^0, t^0) where $u(x^0, t^0) > 0$, the equations are nondegenerate. Degeneracy occurs only at the point (x^0, t^0) where $u(x^0, t^0) = 0$. Because of the degeneracy of the equation, the classical solutions do not exist, so that we must consider the generalized solutions.

The following results were obtained in the study of the Cauchy problem and the first boundary value problem of the above equations, for the one space variable case.

Oleinik, Kalasinkov and Czou [1] treated the Cauchy problem and first boundary value problem of $u_t = [\varphi(x,u)]_{xx}$ where $\varphi(x,0) = \varphi'_u(x,0) = 0$.

Gilding and Petelier [2] studied the Cauchy problem of $u_t = (u^m)_{xx} + (u^n)_x$ (m>1 , m , n , are constants). Under the restriction $n \le \frac{1}{2}(m+1)$, the generalized solutions were proved to be unique, while for the existence of generalized solution this restriction was not needed.