FOURIER TRANSFORM OF SURFACE CARRIED MEASURES.

M. G. Cowling

and

G. Mauceri

This note, which reports on results from [3], is concerned with estimates of the decay of the Fourier transform of measures supported on hypersurfaces of vanishining curvature. Let S be a hypersurface in $\mathbb{R}^{n+1}$ with Gaussian curvature K and area measure dS. Let $w \in C_c^\infty(S)$. We are seeking estimates of the Fourier transform $\widehat{d\mu}$ of the finite Borel measure $d\mu = wdS$ or, more generally, of the measures $d\mu_\alpha = |K|^\alpha w \, dS$, for $\alpha \geq 0$. Such estimates are important in a number of problems, such as counting lattice points inside dilates of S, proving a priori inequalities for maximal averages of functions over dilates and translates of S [2] [10], and in the study of certain operators related to hyperbolic differential operators [8].

The problem of estimating $\widehat{d\mu}$ has a long history. The one dimensional case, i.e. when S is a curve in $\mathbb{R}^2$, has been investigated by van der Corput for number theoretical reasons. More recently estimates for higher dimensional hypersurfaces have been given by Hlwaka [5], Herz [4], Littman [7], Randol [9] and Svensson [12] . If the Gaussian curvature of S does not vanish on the support of w, the method of stationary phase [6] applied to the oscillatory integral