

TRINITY ... A TALE OF THREE CARDINALS

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Dedicated to Igor Kluvanek

1. INTRODUCTION

In this paper we discuss three cardinal numbers associated with a topological group G : the weight of G , $\omega(G)$, the local weight, $\omega_0(G)$, and $\theta(G)$, the least cardinal of a family of open sets whose intersection is a singleton. It is clear that $\theta(G) \leq \omega_0(G) \leq \omega(G)$. We give necessary and sufficient conditions for $\theta(G) = \omega_0(G) = \omega(G)$. In particular they are equal for all σ -compact locally compact Hausdorff groups.

The following notation will be used throughout the paper. If G is a topological group, we denote

- (a) the minimal cardinality of a family of open sets having as intersection the identity, 1 , in G by $\theta(G)$;
- (b) the minimal cardinality of an open basis for G at 1 by $\omega_0(G)$;
- (c) the minimal cardinality of an open basis for G by $\omega(G)$.

If H is a topological subgroup of G , we write $H \leq G$.

Note that if $H \leq G$, then $\theta(H) \leq \theta(G)$, $\omega_0(H) \leq \omega_0(G)$, and $\omega(H) \leq \omega(G)$.