STABLE HARMONIC MAPS WITH VALUES IN $\mathbb{P}^m\mathbb{C}$

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0. INTRODUCTION

Let $f : (M,g) \to (N,h)$ be a stable harmonic map between Kähler manifolds: examples of such maps are holomorphic or antiholomorphic maps and it is important to know when this is the only possible case.

We present here some results on the complex analytic character of stable harmonic maps in the special case where N is the complex projective space $\mathbb{P}^m(\mathbb{C})$ equipped with the Fubini-Study metric and M is low-dimensional.

The results described below start from an investigation of the second variation formula of the energy of f and the information one can get about $\operatorname{Im} f$, using the special form of the curvature tensor of $\mathbb{P}^m(\mathbb{C})$. In the case the differential f_* is generically injective it turns out that f induces, locally on M, an integrable almost complex structure compatible with the metric; i.e. a local complex section of the Twistor Space $Z(M)$ over M. An analysis of the number of such sections allows us to draw conclusions.

These results have been announced in [2]; details and proofs will appear in a subsequent paper.

1. LOCAL CONSEQUENCES OF STABILITY

Let (M,g) be a Riemannian oriented compact manifold and let $N = G/H$ be a compact irreducible Hermitian symmetric space equipped with complex structure tensor $\hat{J}$ and canonical connection; let