REGULARITY THEOREMS FOR ELLIPTIC EQUATIONS WITH NON-SMOOTH COEFFICIENTS

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0. PRELIMINARIES

We are concerned with the elliptic equation

$$(1) \qquad Lu(x) = \sum_{|\alpha| \leq m} a_\alpha(x) \partial^\alpha u(x),$$

where the a_α's are not infinitely differentiable but merely are locally in some Besov space $B^s_{p,q}$ or Triebel space $F^s_{p,q}$. Hereafter we assume that all functions and distributions are defined on $\mathbb{R}^n$. As

$$Lu(x) = \tau(x,D)u(x) = (2\pi)^{-n} \int \tau(x,\xi) e^{ix\cdot\xi} \hat{u}(\xi) d\xi, \quad u \in S$$

where

$$(2) \qquad \tau(x,\xi) = \sum_{|\alpha| \leq m} a_\alpha(x)(i\xi)^\alpha,$$

and

$$\hat{u}(\xi) = \int e^{-ix\cdot\xi} u(x) dx$$

is the Fourier transform of u, one is led to study pseudo-differential operators (ψdos) whose symbols $\sigma(x,\xi)$ (not necessarily of the form (2)) are not smooth in x. In fact, motivated by applications to equation (1), we proved in [Bul] the following result.