SOLVABILITY OF DIFFERENTIAL OPERATORS II: SEMISIMPLE LIE GROUPS

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1. INTRODUCTION

Let G be a noncompact, connected, semisimple Lie group with finite centre and P a differential operator on G. P is <u>left invariant</u> if for all $g \in G$ and for all $f \in C^\infty(G)$, $PL_g f = L_g Pf$, where L_g denotes left translation. Similarly one defines <u>right invariance</u> and <u>bi-invariance.</u>

Elements X of the Lie algebra $\underline{\underline{g}}$ act on $C^\infty(G)$ by

$$(Xf)(g) = \frac{d}{dt}\Big|_{t=0} f(g \exp tX)$$

These define left invariant operators and in fact every left invariant operator is obtained by the extension of this map to the universal enveloping algebra $U(\underline{\underline{g}}_C)$. The bi-invariant operators correspond to the centre $Z(\underline{\underline{g}}_C)$ of $U(\underline{\underline{g}}_C)$.

DEFINITION A differential operator P has <u>fundamental solution</u> E if E is a distribution (in $\mathfrak{D}'$) such that $PE = \delta_e$.

We say that P is <u>locally solvable</u> if $PC^\infty(V) \supseteq C_c^\infty(V)$ for some neighbourhood V of the identity, and P is <u>globally solvable</u> if $PC^\infty(G) = C^\infty(G)$.

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