A NEW APPROACH TO THE SCHAUDER ESTIMATES FOR LINEAR ELLIPTIC EQUATIONS

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In this talk we describe a relatively simple approach to the Schauder estimates for general elliptic systems of the type considered by Douglis and Nirenberg [1]. Our method, as presented in the lectures [6], requires neither preliminary singular integral estimates as in [1] nor auxiliary existence results as in those proposed by Campanato, (see [2]), and Safonov [5]. Instead our procedure involves the direct deduction of the Hölder estimates from the corresponding L^2 estimates for the constant coefficient case, by means of mollification. It is also readily extended to more general classes of operators. In the special case of a single second order equation, the classical mean value inequality for subharmonic functions can be used in place of the L^2 estimates.

To illustrate the technique, we confine attention here to elliptic systems of the form,

$$(1) \qquad L^i u = \sum_{j=1}^{N} \sum_{\substack{|\alpha|=s_i, \\ |\beta|=t_j}} D^\alpha \left[a_{\alpha\beta}^{ij} D^\beta u_j \right] = \sum_{|\alpha|=s_i} D^\alpha f_\alpha^i$$

with complex valued coefficients $a_{\alpha\beta}^{ij}$ and inhomogeneous terms f_α^i, $i,j = 1,\ldots N$, $|\alpha| = s_i$, $|\beta| = t_j$, in $C^\gamma(\mathbb{R}^n)$, $0 < \gamma < 1$, with s_i, t_i, $i = 1,\ldots N$ non-negative integers. The full generality of [1] may be recovered by some modification together with the standard interpolation inequalities, ([1],Section 2,[3],Section 6.8). Indeed we