ESTIMATES FOR LINEAR SYSTEMS OF OPERATOR EQUATIONS

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1. INTRODUCTION

This is a description of joint work(*) with Alan McIntosh and Werner Ricker of Macquarie University.

Throughout, X and Y denote (complex) Banach spaces. The space of bounded (linear) operators from X to Y, provided with the operator norm, is denoted $L(X,Y)$ and $L(X) = L(X,X)$. The Taylor spectrum of a commuting m-tuple $\underset{\sim}{S} = (S_1,\dots,S_m)$ in $L(X)^m$ is denoted $Sp(\underset{\sim}{S})$ or $Sp(S_1,\dots,S_m)$ or $Sp(\underset{\sim}{S},L(X))$ (see Taylor [9]).

We consider the following linear system of equations

$$(1.1)\qquad \sum_{j=1}^{n} A_{ij}QB_{ij} = U_i \quad \text{for} \quad 1 \leqq i \leqq m.$$

Here and elsewhere, $\underset{\sim}{A} = (A_{ij}) \in L(X)^{mn}$, $\underset{\sim}{B} = (B_{ij}) \in L(Y)^{mn}$, $1 \leqq i \leqq m$, $1 \leqq j \leqq n$, and $\underset{\sim}{A}$, $\underset{\sim}{B}$ are commuting mn-tuples. Moreover, $\underset{\sim}{U} = (U_1,\dots,U_m) \in L(Y,X)$ is given and an operator $Q \in L(Y,X)$ satisfying (1.1) is to be determined. We will order mn-tuples such as $\underset{\sim}{A} = (A_{ij})$ or $x = (x_{ij}) \in \mathbb{C}^{mn}$, $1 \leqq i \leqq m$, $1 \leqq j \leqq n$, lexicographically from the left. So, $x = (x_{11}, \dots, x_{1n}, x_{21}, \dots, x_{2n}, \dots, x_{m1}, \dots, x_{mn})$.

For $m > 1$, the system (1.1) is overdetermined and it is readily seen that a necessary condition for the solubility of (1.1) is the following

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