ON ISOMORPHISMS OF ALGEBRAS OF OPERATORS

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The starting point of the investigations described here is Pontryagin duality. If G is a locally compact abelian group, and $\hat{G}$ is its character group, i.e. the group $\mathrm{Hom}(G,\mathbb{T})$, then $\hat{\hat{G}} = G$, and $G \to \hat{G}$ is a contravariant functor on the category LCAG of locally compact abelian groups, with morphisms being continuous homomorphisms. This theorem, together with its analytic versions, concerning the Fourier transformation, inspired substantial research on general locally compact abelian groups, and at the same time begged the question of what analogues hold for other groups. It is generally accepted that the right answer to this question involves the continuous unitary representations of G, as the natural analogue of $\mathrm{Hom}(G,\mathbb{T})$, but the structures involved are more complicated.

To describe some further developments, a number of group algebras and spaces should be described. For a general locally compact group, $\hat{G}$ denotes the space of continuous irreducible unitary representations π of G on a Hilbert space, H_π, modulo unitary equivalence. If G is abelian, this coincides with the space $\hat{G}$ described before, but the fact that $\hat{G}$ is a group is lost unless one considers tensor products of representations (corresponding to multiplication of characters), which is