EVOLUTION OPERATORS OF PARABOLIC EQUATIONS IN CONTINUOUS FUNCTION SPACE

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1. INTRODUCTION

Let

$$(P)\quad \begin{cases} \partial u/\partial t + \sum_{|\alpha|\le 2m} a_\alpha(t,x)D^\alpha u = f(t,x) & \text{in } (0,T] \times \Omega \\ \sum_{|\beta|\le m_j} b_{j\beta}(t,x)D^\beta u = 0 & \text{on } (0,T] \times \partial\Omega,\ j = 1,\cdots,m \\ u(0,x) = u_0(x) & \text{in } \Omega \end{cases}$$

be the initial value problem of a parabolic partial differential equation in a (bounded or unbounded) region Ω in $\mathbb{R}^n$. This Note studies the construction of an evolution operator (fundamental solution) for (P) in the continuous function space $\mathscr{C}(\overline{\Omega})$ on $\overline{\Omega}$. In the L_p $(1 < p < \infty)$ space case the construction has been studied by several authors, including Kato et al.[1], Tanabe [4] and Yagi [6]. Recently Tanabe [8] and his student Park [2] showed existence of the evolution operator for (P) even in a "worse" function space $L^1(\Omega)$ (recall that there is no a priori estimate for elliptic operators in L^1 space). We are then interested to work in another "worse" function space $\mathscr{C}(\overline{\Omega})$.

For $0 \le t \le T$ let $A(t)$ denote the operator