A DUALITY THEOREM FOR CROSSED PRODUCTS BY NONABELIAN GROUPS

Iain Raeburn

Let $\alpha : G \to \mathrm{Aut}\, A$ be an action of a locally compact group G on a C*-algebra A. When G is abelian, there is a canonical dual action $\hat{\alpha}$ of the dual group $\hat{G}$ on the crossed product $A \times_\alpha G$, and the Takai duality theorem asserts that the second crossed product $(A \times_\alpha G) \times_{\hat{\alpha}} \hat{G}$ is isomorphic to the tensor product $A \otimes \mathcal{K}(L^2(G))$ of A with the algebra $\mathcal{K}(L^2(G))$ of compact operators on $L^2(G)$. The usual proof of this theorem [5,8] uses spatial techniques, but we have recently given a new proof in which we exploit the universal properties of crossed products, and which we hope is a bit more transparent [7].

Imai and Takai used essentially the same spatial techniques to obtain a duality theorem for actions of nonabelian groups [1]. They replaced the dual action of $\hat{G}$ by a "coaction" of G, and defined all their crossed products spatially, so for non-amenable G their theorem concerns the reduced crossed product $A \times_{\alpha,r} G$ rather than the full one $A \times_\alpha G$. Here we shall show that the approach of [7] can also be adapted to the case of nonabelian groups, where it gives a duality theorem for the full crossed product.

We start by describing our notion of coaction, which is slightly different from the normal one: usually a coaction of G on A is a homomorphism of A into $M(A \otimes_{\min} C_r^*(G))$, whereas ours will be a homomorphism of A into $M(A \otimes_{\max} C^*(G))$. We have deliberately chosen to use the full group C*-algebra and the maximal tensor product because we are stressing universal properties rather than spatial ones. As a