CENTRALLY TRIVIAL AUTOMORPHISMS OF C^*-ALGEBRAS

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We continue our study of central sequences in, and automorphisms of separable C^*-algebras begun in [7]. We would like to attempt, as far as possible, to follow the plan of attack developed by A. Connes in [2,3] for von Neumann algebras. Unfortunately, very few of the general ideas survive in the C^*-algebra setting. The main reason for this is the overabundance of nontrivial central sequences in a general separable C^*-algebra. Despite this, we are able to sufficiently analyze some large classes of C^*-algebras so that those automorphisms which behave trivially on central sequences can be computed. Complete proofs and more detailed examples will appear elsewhere.

§ 1. Preliminaries

Let A denote a separable unital C^*-algebra over the complex numbers. Let AutA denote the group of all *-automorphisms of A, and InnA the normal subgroup of all inner automorphisms. Let $\varepsilon : \mathrm{AutA} \to \mathrm{AutA/InnA} = \mathrm{OutA}$ be the quotient.

A central sequence in A is a bounded sequence $\{x_n\}$ of elements of A with the property that $[x_n,a] = x_n a - a x_n \to 0$ in norm for each $a \in A$. A uniformly central sequence is a bounded sequence $\{x_n\}$ for which the operators (on A) $\mathrm{ad}x_n(\cdot) = [x_n,\cdot]$ converge to 0 in norm. A central sequence $\{x_n\}$ is called hypercentral if $\|[x_n,y_n]\| \to 0$ for every central seqeunce $\{y_n\}$ of A. A central sequence $\{x_n\}$ is called trivial if there is a sequence $\{\lambda_n\}$ of central elements in A so that $\|x_n-\lambda_n\| \to 0$. It is evident that any trivial sequence is uniformly central and any uniformly central