ERGODIC MEASURES FOR THE ACTIONS OF DENSE SUBGROUPS

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1. INTRODUCTION

A construction for singular measures quasi-invariant and ergodic for the irrational rotation T_α by α on the circle was first given by Keane in [5]. It involved using the continued fraction expansion of α to identify T_α with the odometer action on a subset of an infinite product space. Subsequently Katznelson and Weiss [4] obtained a general method for constructing uncountably many mutually singular quasi-invariant measures ergodic with respect to a general homeomorphism T on a compact metric space, provided that T has a recurrent point.

In a recent paper [6], we have shown that it is possible to use a Riesz product technique to obtain measures of the kind that Keane produced; though, being Riesz products these measures are more susceptible to control of their Fourier-Stieltjes transform. In particular such measures can be chosen to have their Fourier-Stieltjes transform vanishing at infinity. This property was required in connection with the study of non-monomial representations of the discrete Heisenberg group (cf. [1]).

The idea of using Riesz products to obtain ergodicity was first introduced by Gavin Brown in [2]. He was, however, only able to deal with subgroups in which every element has finite order. Nevertheless it was a modification of his technique which was used in [6] to obtain measures ergodic for the irrational rotation. In this paper we generalise the technique to apply to any countable dense subgroup of a compact metric abelian group.